JUNIOR PROBLEMS

Solutions to the problems stated in this issue should arrive before June 19, 2017
.

Proposals

61. *Proposed by Nguyen Viet Hung, Hanoi University of Science, Vietnam.* Given a tetrahedron $A_1A_2A_3A_4$ with the volume V, let I and r be incenter and inradius, respectively. Denote by S_i the area of triangle opposite to vertex $A_i (i = 1; 2; 3; 4)$. Prove that

$$\sum_{n=1}^{4} S_i IA_i^2 = \frac{2rS_1S_2S_3S_4}{9V^2} \sum_{1\le i<j\le 4} A_iA_j \sin\angle(A_i, A_j),$$

where $\angle(A_i, A_j)$ is the dihedral angle at edge A_iA_j.

62. *Proposed by Daniel Sitaru, Mathematics Department, Colegiul National Economic Theodor Costescu, Drobeta Turnu - Severin, Mehedinti, Romania.* Let be $A', A'' \in (BC); B', B'' \in (AC); C', C'' \in (AB)$ in ΔABC such that $AA' \cap BB' \cap CC' \neq \emptyset$ and $AA'' \cap BB'' \cap CC'' \neq \emptyset$. Prove that

$$\frac{27[A'B'C']}{[A''B''C'']} \le \left(\frac{BA'}{BA''} + \frac{CB'}{CB''} + \frac{AC'}{AC''}\right)^3,$$

where $[ABC]$ is area of triangle ABC.

63. *Proposed by Leonard Giugiuc, National College Traian, Drobeta Turnu Severin, Romania.* Let $a, b, c \in \mathbb{R}$. Prove that

$$9\sqrt{2}(ab(a-b) + bc(b-c) + ca(c-a)) \le \sqrt{3}\left((a-b)^2 + (b-c)^2 + (c-a)^2\right)^{\frac{3}{2}}.$$

64. *Problem proposed by Arkady Alt, San Jose, California, USA.* Let $\Delta(x, y, z) := 2(xy + yz + xz) - (x^2 + y^2 + z^2)$ and let a, b, c be sidelengths of a triangle with area F. Prove that

$$\Delta\left(a^3, b^3, c^3\right) \le \frac{64F^3}{\sqrt{3}}.$$

65. *Proposed by Dorlir Ahmeti, University of Prishtina, Department of Mathematics, Republic of Kosova.* Find all function $f : \mathbb{N} \to \mathbb{N}$ such that $mf(n) + f(m)$ is divisible by $f(m)(f(n) + 1)$ for all $m, n \in \mathbb{N}$.

Solutions